1. Introduction.—If $V$ is a non-singular (irreducible) algebraic surface, we can define its “good Betti numbers” in the following way. We take both $B_2$ and $B_4$ to be equal to 1. Also, we take $B_1$ and $B_3$ to be the twice of the dimension of the Picard variety of $V$. Finally, we define $B_2$ so that we get

$$
\Sigma_i(-1)^iB_i = \chi \quad (= \text{Euler number of } V).
$$

The definition is not vacuous, because $\chi$ has an independent meaning. In fact $\chi - 4$ is known as Zeuthen–Segre’s invariant of $V$.\footnote{Theorem} We observe, however, that this definition leaves uncertain whether $B_2$ is positive or not. In this short note, we shall outline a proof of the theorem stating that our two-dimensional Betti number $B_2$ is at least equal to the Picard number $\rho$ of $V$. This fact can be looked upon as giving a precise upper bound to the Picard number. It might be worthwhile to investigate the possibility of applying our method to absolute surfaces in connection with the rank problem of elliptic curves or, more generally, of Abelian varieties.

2. Preliminary Observations.—Let $\omega$ be a linear differential on $V$ different from 0. Then, we can associate an 0-cycle

$$
\langle \omega \rangle = \Sigma_{P \in V} m_P(\omega) \cdot P
$$

to $\omega$ in the following way. We take local parameters $t_1, t_2$ of $V$ at $P$ and we write $\omega$ in the form $f \cdot (h_1dt_1 + h_2dt_2)$ by three functions $f, h_1, h_2$ on $V$ such that $h_1, h_2$ are holomorphic and have no zero curve in common at $P$. Then, the multiplicity of $P$ in the intersection of divisors $(h_1), (h_2)$ of $h_1, h_2$ depends only on $P$ and $\omega$. This we take as $m_P(\omega)$. On the other hand, let $\Omega$ be a double differential on $V$ different from 0. We shall use the standard notation $(\omega), (\Omega)$ to denote divisors of the differentials $\omega, \Omega$. Then, we can consider $\langle \omega \rangle + (\omega) \cdot (\Omega) - (\omega) \cdot (\omega)$ as defining a rational equivalence class on $V$.\footnote{Note} The point is that this class is intrinsic, i.e., it does not depend on the choice of $\omega$ and $\Omega$. The proof is left to the reader. Actually, we can show that this class is the second canonical class of $V$. At any rate, if we take its degree, i.e., if we pass to algebraic equivalence, we get a relative invariant $\chi$. There is a classical way of calculating this number.

We have shown elsewhere\footnote{Elsewhere} that we can find a function $f$ on $V$ with values in a numerical straight-line $D$ such that (i) every fiber $C_u = f^{-1}(u)$ is absolutely irreducible; (ii) there exists a finite number of points, say $a_1, \ldots, a_n$, on $D$ such that the fibers become singular only at these points; (iii) any two fibers are transversal to each other at a finite number of points, say, $b_1, \ldots, b_g$. We note that, if $d_1, \ldots, d_n$ are the double points of fibers over $a_1, \ldots, a_n$, we have $\langle df \rangle = \Sigma d_i + \Sigma b_j$. Moreover, if we denote the constant arithmetic genus of $C_u$ by $g$, the degree of the rational equivalence class $(\Omega) + (f) - (f)$ is $2g - 2$. Since we also have $(df) = -2(f)$, we get

$$
\chi = \alpha - \beta - 4g + 4.
$$
We need another observation. Let $k$ be an arbitrary algebraically closed field of definition of $V$ and $f$. Let $u$ be a generic point of $D$ over $k$ and let $J$ be the Jacobian variety of $C_u$ defined over $k(u)$. Since the $b_l$'s are rational over $k$, we can normalize the canonical function $\varphi$ so that we have $\varphi(b) = 0$ for $b = b_l$ say. We shall denote by $M$ the group of rational points of $J$ with reference to $k(u)$. The group $M$ contains a subgroup $N$ defined in the following way. Let $X$ be a divisor of $V$ which is algebraically equivalent to zero. If $X$ is rational over $k$, the image point in $J$ of the intersection $X \cdot C_u$ is an element of $M$. The group $N$ consists of elements of this nature. We note that $N$ can be considered as the group of rational points over $k$ of the Picard variety of $V$. In particular, it is divisible. Therefore, we have the splitting $M = N \times (M/N)$ and, by Néron's result, the factor group $M/N$ is finitely generated. Moreover, the rank of its free part is equal to $\rho + \beta - 2$. In fact, let $V^*$ be the graph in the product $V \times D$ of the function $f$. Then $V^*$ is a quadratic transform of $V$ centered at $b_1, \ldots, b_l$. In particular, it is non-singular and the Picard number is $\rho + \beta$. Let $X^*$ be a divisor of $V^*$ rational over $k$ such that the intersection $X^* \cdot (C_u \times u) = \Sigma n_i(P_i \times u)$ is of degree zero. Then, we have $\varphi(X^*) = \Sigma n_i \varphi(P_i) = 0$ if and only if $X^*$ is linearly equivalent on $V^*$ to an integer multiple of $C_u \times b$. Moreover, every element of $M$ can be written in the form $\varphi(X^*)$. Therefore, the rank of the free part of $M/N$ is equal to the Picard number of $V^* - 2$, which is $(\rho + \beta) - 2$ as asserted.

3. Cohomological Calculation Using Abstract Vanishing Cycle Theory.—We take a prime number $p$ different from the characteristic of $k$. Let $x$ be a point of $J$ such that $px$ is in $M$. We shall denote by $L$ the group of such points. Then, the kernel of the epimorphism of $L$ to $M$ defined by $x \mapsto px$ is the group of points of $J$ of order $p$, which we shall denote by the standard notation $\mathfrak{r}J$. Since $\mathfrak{r}J$ is of order $p^2$, if $p$ does not divide the order of the torsion part of $M/N$, we have $\mathfrak{r}J \cap M = pN$. On the other hand, we know that $k(u)(L)$ is a tamely ramified finite Galois extension of $k(u)$ ramified at $a_1, \ldots, a_{\alpha}$. Therefore, by a general result of Grothendieck we can state the following lemma:

**Lemma.** The Galois group $G$ of $k(u)(L)$ over $k(u)$ is generated by some generators $\sigma_1, \ldots, \sigma_{\alpha}$ of inertia groups over $a_1, \ldots, a_{\alpha}$ satisfying $\sigma_{\alpha} \cdots \sigma_1 = 1$.

Now, let $v_i$ be the group of vanishing points which correspond to the inertia group generated by $\sigma_i$. It is a cyclic subgroup of $\mathfrak{r}J$ of order $p$ and, if $x$ is an arbitrary element of $L$, we know that $x - \sigma_i x$ is in $v_i$. Let $(x, y) \rightarrow e(x, y)$ be the skew-symmetric non-degenerate bilinear form defined on $\mathfrak{r}J$ with values in the cyclic group of $p$th roots of unity. If $p$ is not in a possible finite set of prime numbers, the induced bilinear form on $\mathfrak{r}N$ is also non-degenerate. We shall, then, show that an element $y$ of $\mathfrak{r}J$ for which $e(x, y) = 1$ for all $x$ in the group generated by $\mathfrak{r}N$ and $v_1, \ldots, v_{\alpha}$ is 0, i.e., these subgroups generate the whole $\mathfrak{r}J$. Let $z$ be an arbitrary element of $\mathfrak{r}J$. Then, we have $e(z - \sigma_i z, y) = 1$ for $i = 1, \ldots, \alpha$, hence $e(x + z, y) = 1$ for every $x$ in $G$. However, this implies $e(x, \sigma^{-1}y - y) = e(\sigma z - z, y) = 1$, hence $e^{-1}y = y$. Therefore, $y$ is in $\mathfrak{r}J \cap M = pN$. Since $e(x, y) = 1$ also for all $x$ in $\mathfrak{r}N$, we get $y = 0$ as asserted. These being remarked, we consider the following exact sequence of $G$-modules:

$$0 \rightarrow \mathfrak{r}J \rightarrow L \rightarrow M \rightarrow 0.$$ 

Then, we get an exact sequence $M \rightarrow M \rightarrow H^1(G, \mathfrak{r}J)$ in which the first homo-
morphism is \( x \mapsto px \) and the second one is the connecting homomorphism \( \mathcal{O}^* \). If we observe that \( G \) is generated by \( \sigma_1, \ldots, \sigma_a \), we can interpret \( \mathcal{O}^* \) in the following way. Define a homomorphism \( \lambda \) of \( L \) to the product \( P = v_1 \times \ldots \times v_a \) by

\[
(x \mapsto (x - \sigma_1 x, \sigma_1 x - \sigma_2 \sigma_1 x, \ldots).
\]

Then \( \lambda \) defines a homomorphism \( \lambda^* \) of \( M \) to \( P/\lambda(pJ) \), and this is substantially \( \mathcal{O}^* \). Therefore \( \text{Im}(\lambda^*) \) is isomorphic to \( M/p(M) \), i.e., to \( (M/N)/p(M/N) \). Hence the rank of \( \text{Im}(\lambda^*) \) is \( \rho + \beta - 2 \). On the other hand, consider the homomorphism \( \mu \) of \( P \) to \( pJ/pN \) defined by

\[
(x_1, \ldots, x_a) \mapsto \Sigma x_i \text{ mod } pN.
\]

We know that \( \mu \) is surjective. Moreover, since we have \( \lambda^* = 0 \), we get an epimorphism \( \mu^* \) of \( P/\lambda(pJ) \) to \( pJ/pN \). Since the kernel of the restriction of \( \lambda \) to \( pJ \) is \( pJ \cap M = pN \), which is of rank \( B_1 \), the rank of \( \lambda(pJ) \) is \( 2g - B_1 \). Therefore, the rank of \( \text{Ker}(\mu^*) \) is \( \alpha - 2(2g - B_1) \). Since \( \text{Im}(\lambda^*) \) is contained in \( \text{Ker}(\mu^*) \), we finally get

\[
\rho + \beta - 2 \leq \alpha - 2(2g - B_1),
\]

and this is another way of writing the inequality \( \rho \leq B_2 \).

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4 Throughout the paper, we refer to Weil, A., "Variétés Abéliennes et Courbes Algébriques" ["Actualités sci. et ind.," No. 1064 (Paris: Hermann & Cie, 1948)] for results on Abelian varieties.

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**NITROGEN FIXATION BY CELL-FREE PREPARATIONS FROM MICROORGANISMS**

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During the past quarter century, repeated attempts have been made to obtain cell-free nitrogen fixation with extracts from a wide variety of biological agents. Probably the first attempt was that of Bach et al. who claimed that cell-free preparations from *Azotobacter chroococcum* fixed sufficient \( \text{N}_2 \) to be detected by Kjeldahl analysis. However, Roberg could not reproduce these results, and Bach com-